Post-quantum zero-knowledge proofs with constant-depth quantum simulators

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Research work under the supervision of Alex B. Grilo & Damien Vergnaud

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Zero-knowledge proof: interactive protocol between a prover $\mathcal{P}$ and verifier $\mathcal{V}$. [GMR85]

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Definition

A **simulator** S for a protocol Π is a polytime probabilistic algorithm with black-box access to $\mathcal{V}$, and which output is indistinguishable from the transcript of a real execution of Π.

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Conjecture

Let $\mathbb{L}$ be a language that admits a ZK protocol with a constant-depth simulator; then, $\mathbb{L} \in \mathbf{BQP}$.[a]

[a] BQP = Bounded-error Quantum Polynomial time

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- Exact definition of black-box zero-knowledge:
  - $\rightarrow$ A ZK black-box protocol $\implies$ there exists a simulator *Sim* such that **for all verifiers**, Output$(Sim(\mathcal{V})) \approx$ Transcript$(\mathcal{P} \leftrightarrow \mathcal{V})$

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$$U'_i = |S_{i+1}\rangle\langle S_i| \otimes U_i + |S_i\rangle\langle S_{i+1}| \otimes U_i^\dagger + \sum_{\substack{x \\ |x\rangle\perp|S_i\rangle \\ |x\rangle\perp|S_{i+1}\rangle}} |x\rangle\langle x| \otimes \mathbb{1}$$

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Transcript$(\mathcal{P} \leftrightarrow \mathcal{V}') \approx$ Transcript$(\mathcal{P} \leftrightarrow \mathcal{V})$ **and** Output$(Sim(\mathcal{V}')) \approx$ Output$(Sim(\mathcal{V}))$

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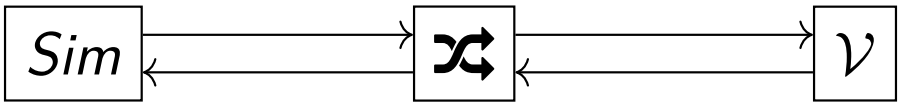

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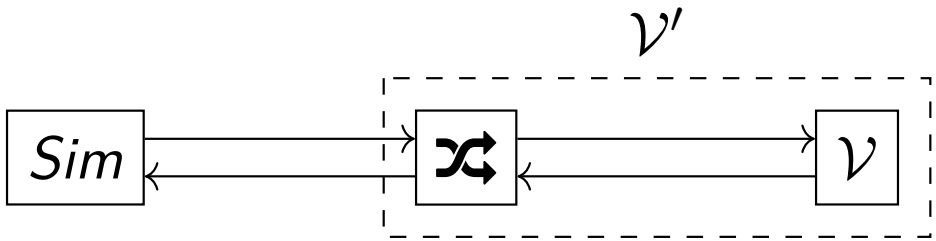

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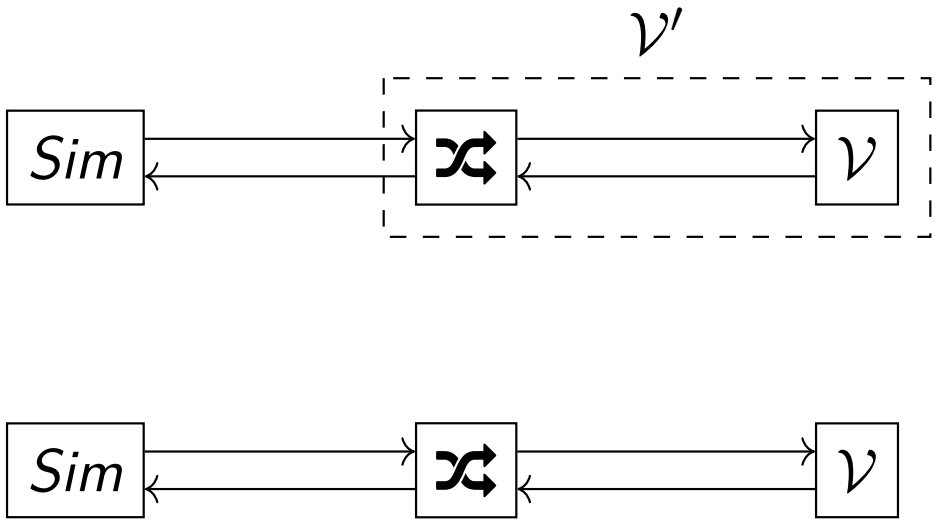

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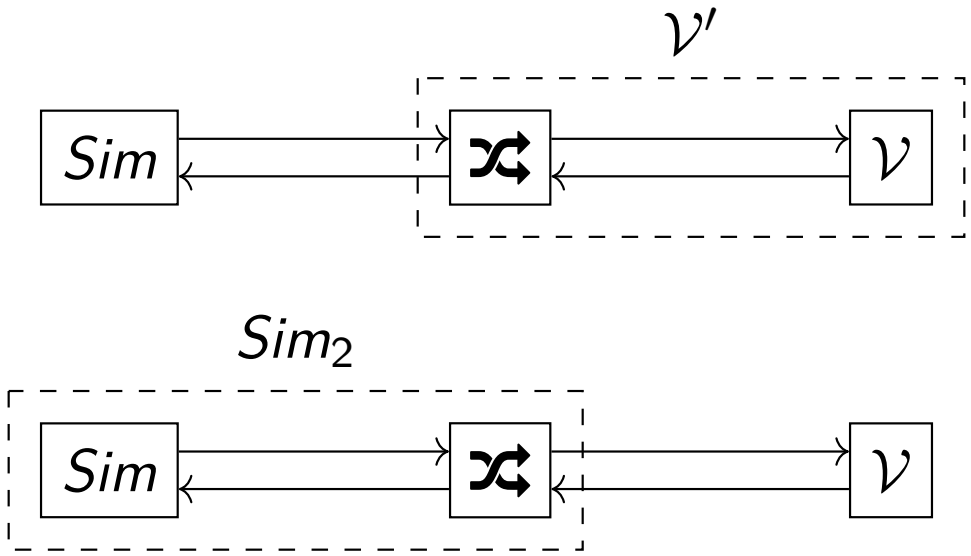

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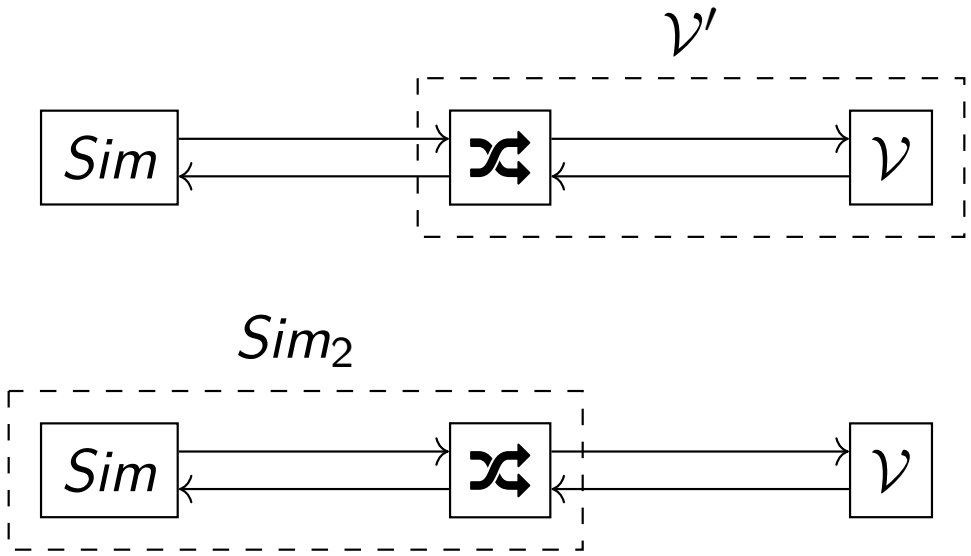

Consequence

$\text{Output}(Sim_2(\mathcal{V})) = \text{Output}(Sim(\mathcal{V}'))$

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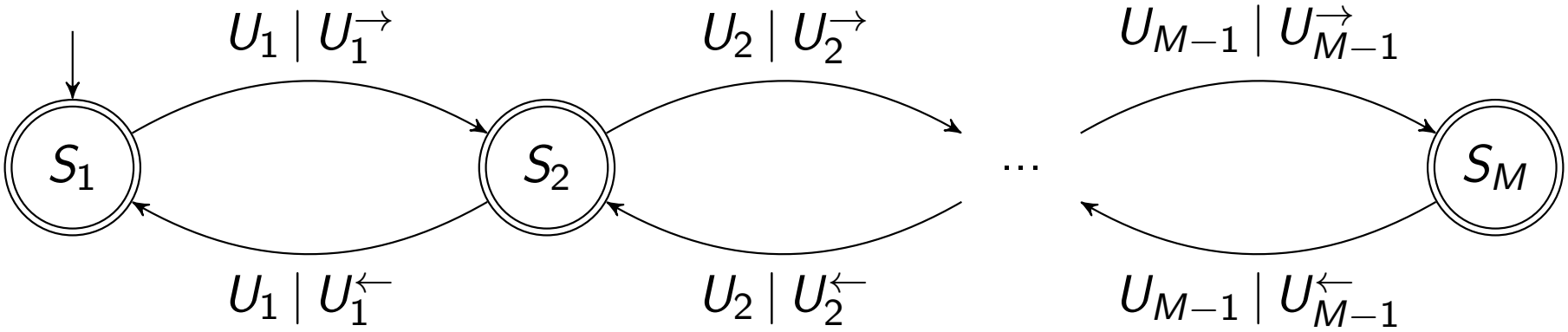

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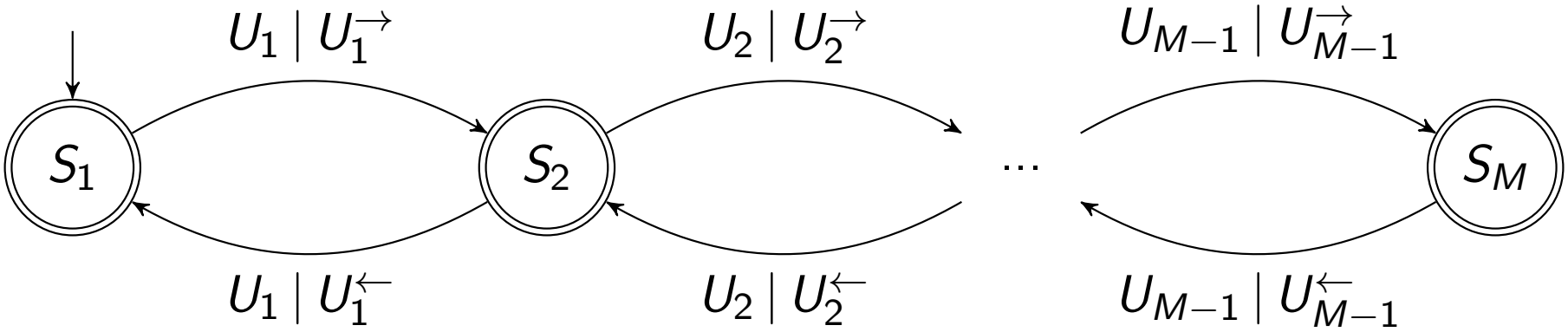

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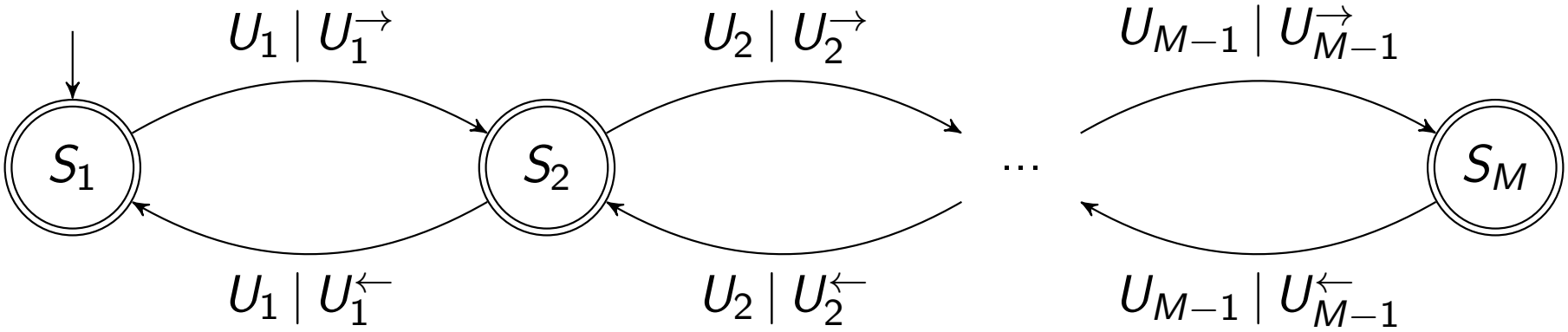

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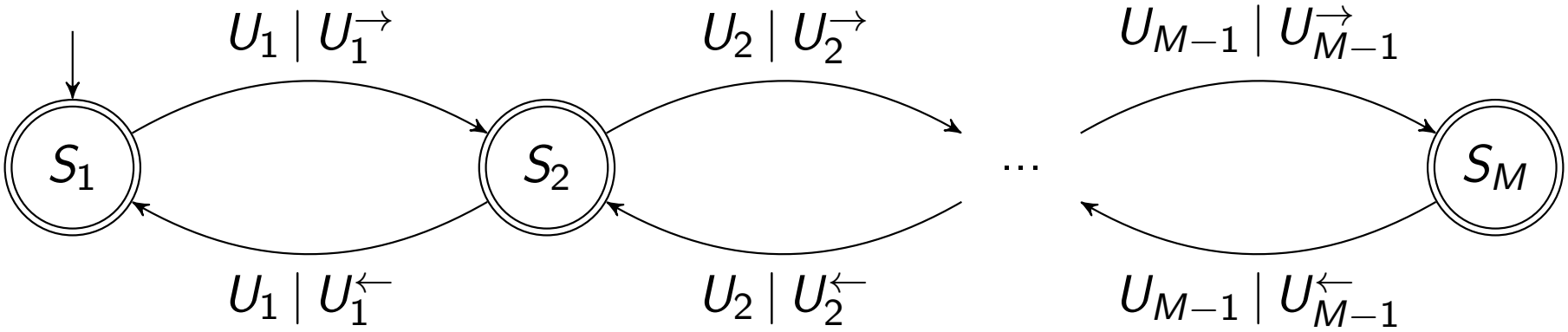

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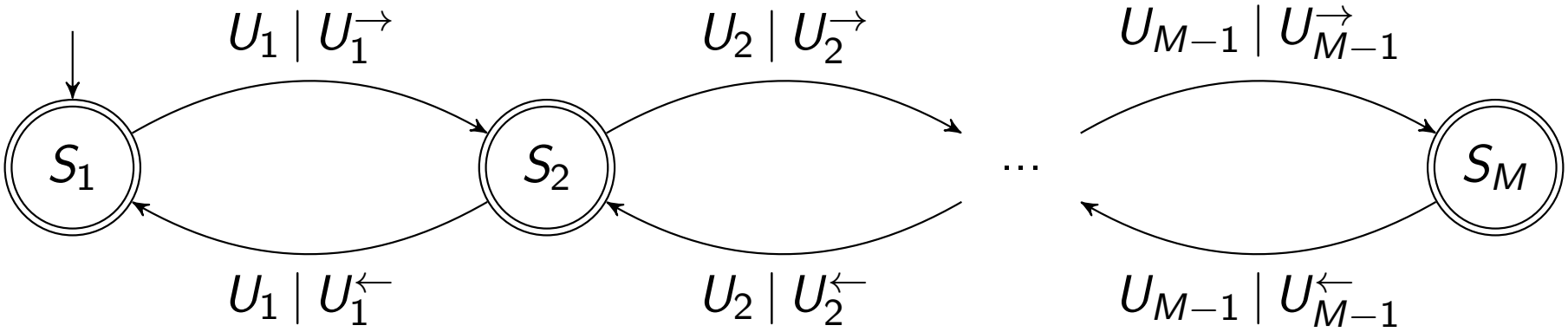

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For both case, we can build $\mathcal{A}$, which takes a quantum coin $|\$\rangle$ as input, and returns $|\$\rangle \otimes |\$\rangle$ with nonnegligible probability.

Contributions: intuition of proof

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- If Sim_3 applies $U_i^{\leftarrow}$: it projects onto $\{|S_{i+1}\rangle\langle S_{i+1}|, \mathbb{1} - |S_{i+1}\rangle\langle S_{i+1}|\}$

Two observations if $U_i^{\leftrightarrows}$ **failed**:

$\rightarrow$ If $U_i^{\leftrightarrows}$ is $U_i^{\rightarrow}$, then the input subspace state $|\Psi\rangle$ can only be $|S_{i+1}\rangle$

$\rightarrow$ If $U_i^{\leftrightarrows}$ is $U_i^{\leftarrow}$, then the input subspace state $|\Psi\rangle$ can only be $|S_i\rangle$

In both subcases, $\mathcal{A}$ can clone any arbitrary quantum coin:
$\mathcal{A}(|\$\rangle \otimes |0\rangle) = |\$\rangle \otimes |\$\rangle$.

Contributions: intuition of the proof (cont'd)

Subcase 1: $U_i^{\leftrightarrows}$ is $U_i^{\rightarrow}$ and failed; therefore, $|\Psi\rangle = |S_{i+1}\rangle$ necessarily.

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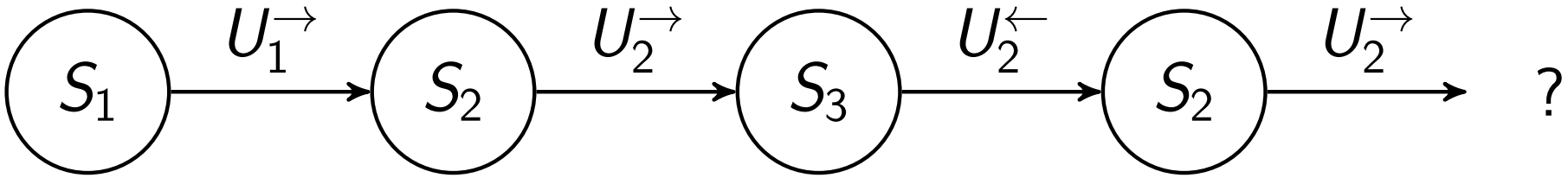

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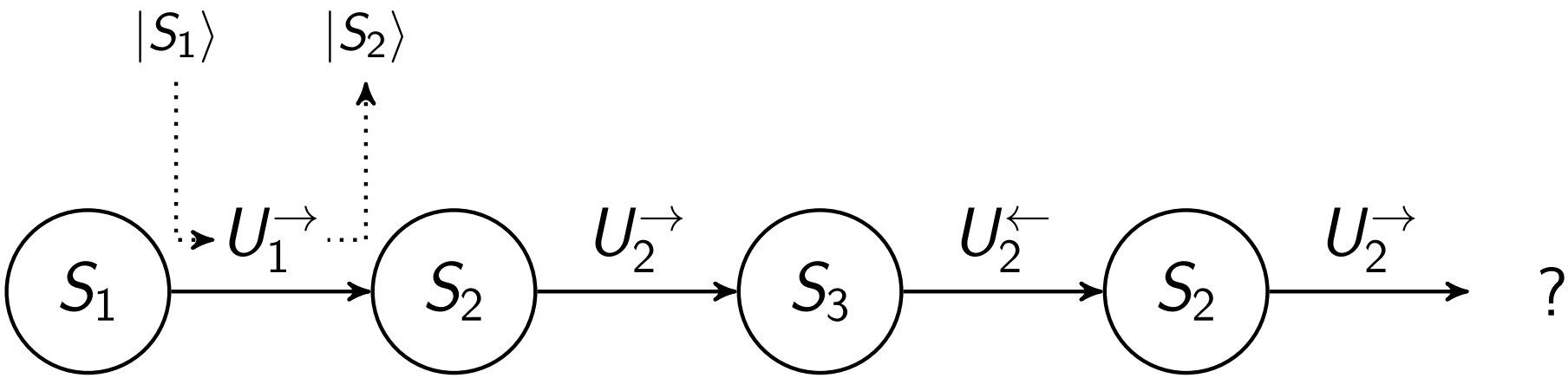

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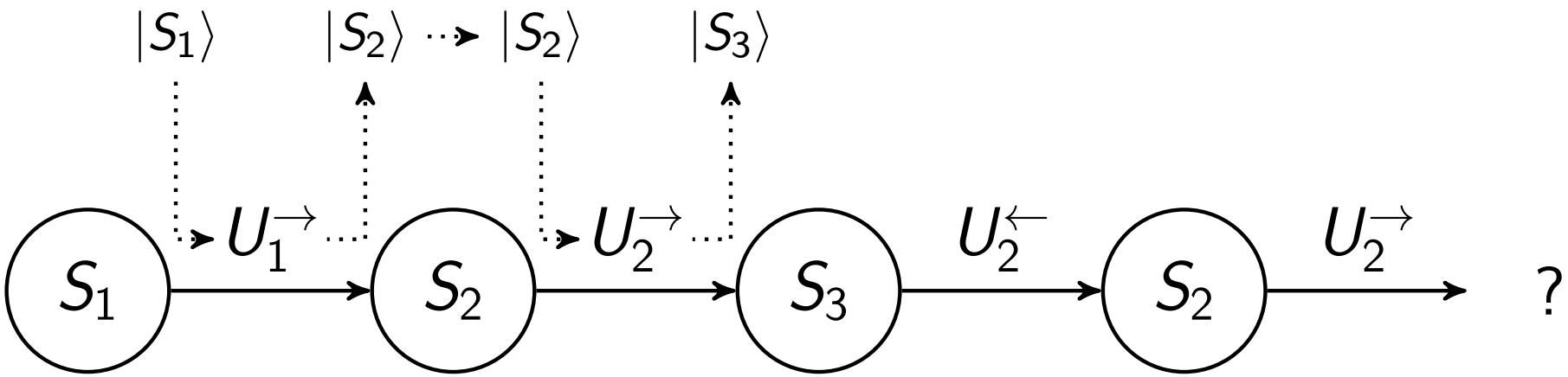

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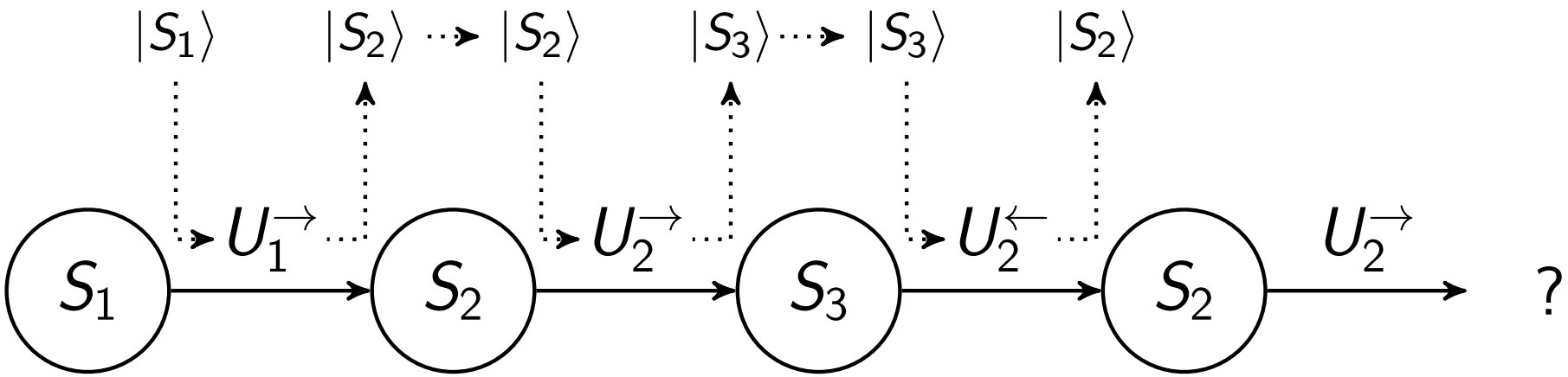

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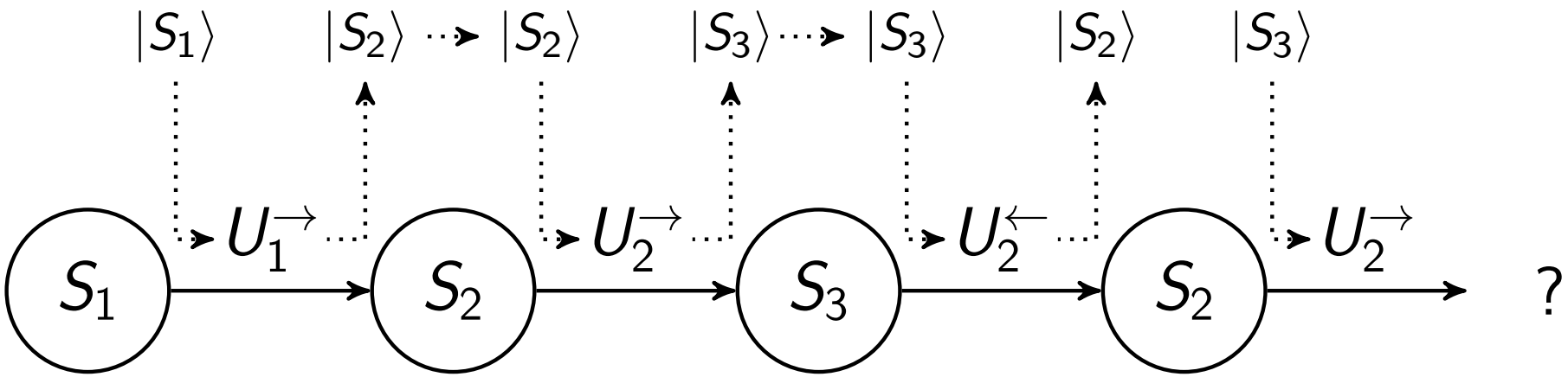

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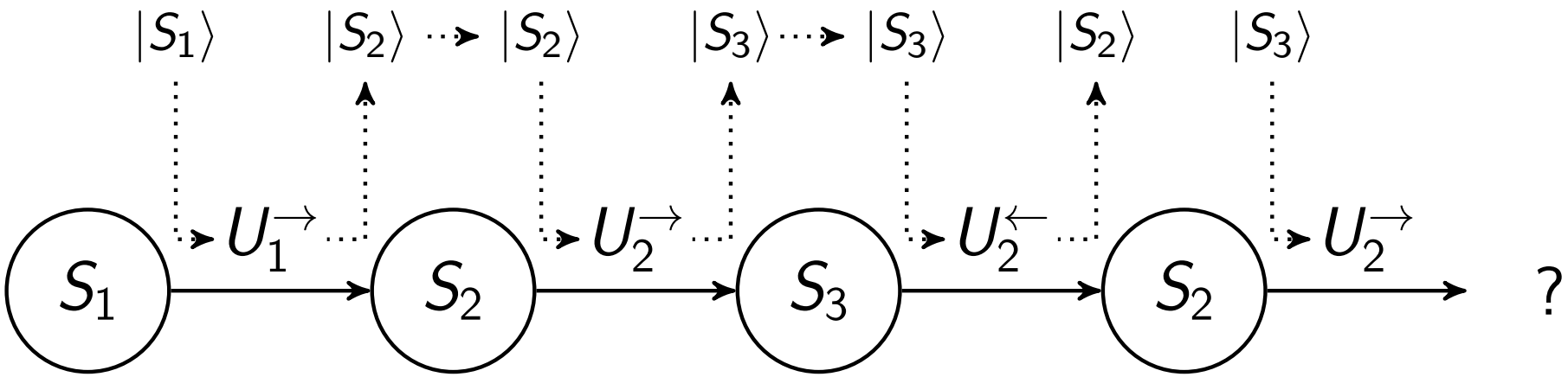

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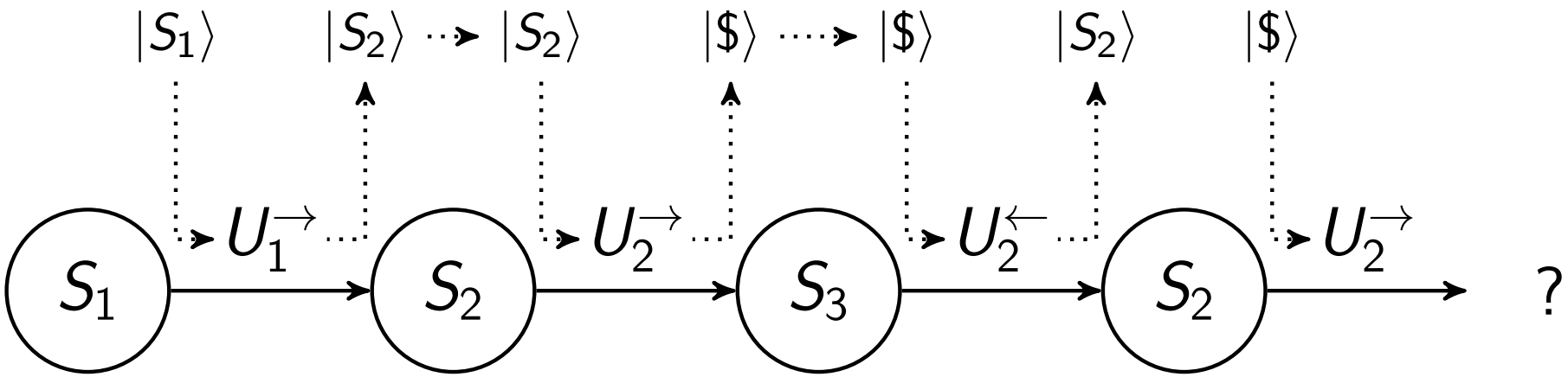

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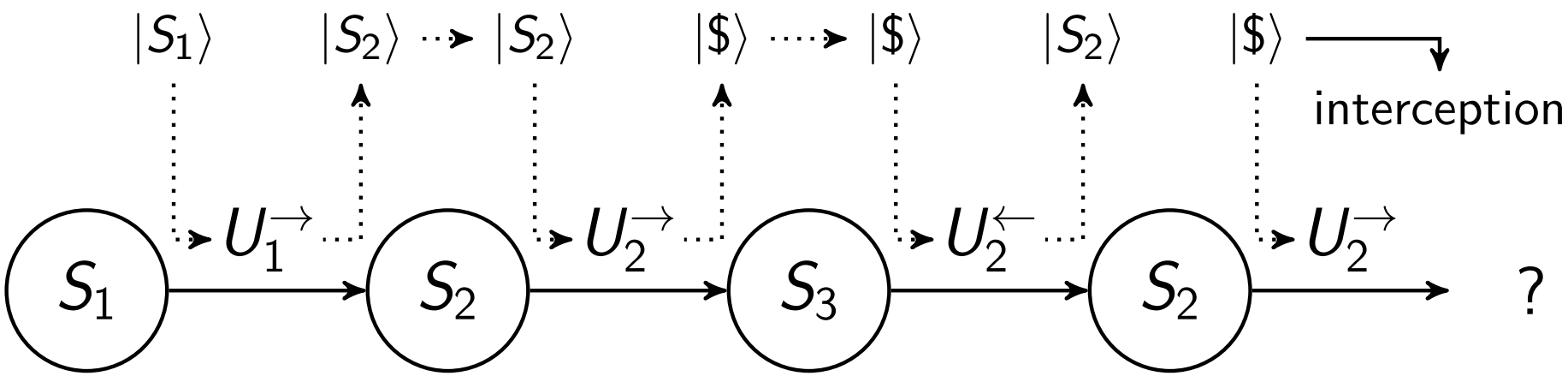

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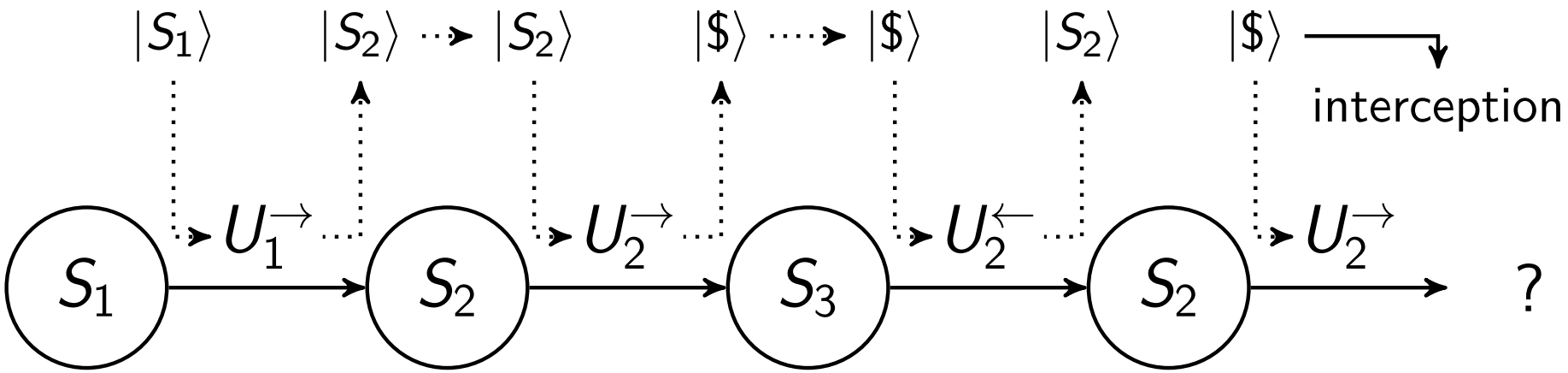

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$\rightarrow$ What $\mathcal{A}$ returns is $|\$\rangle \otimes |\$\rangle$ with probability $1/\text{poly}$.

Contributions: properties of Sim_3

Consequence

If $Sim_3(\mathcal{V})$ is distinguishable from $Sim_2(\mathcal{V})$, we can easily clone a quantum coin. **Absurd by [AC12], therefore Lemma 2 is true.**

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$\text{Output}(Sim_3(\mathcal{V})) \approx \text{Transcript}(\mathcal{V} \leftrightarrow \mathcal{P})$.

$\rightarrow$ All sequential simulators respect the constraints imposed by FSM + Sim_3.

$\rightarrow$ What about parallel ones?

Contributions: parallel simulator

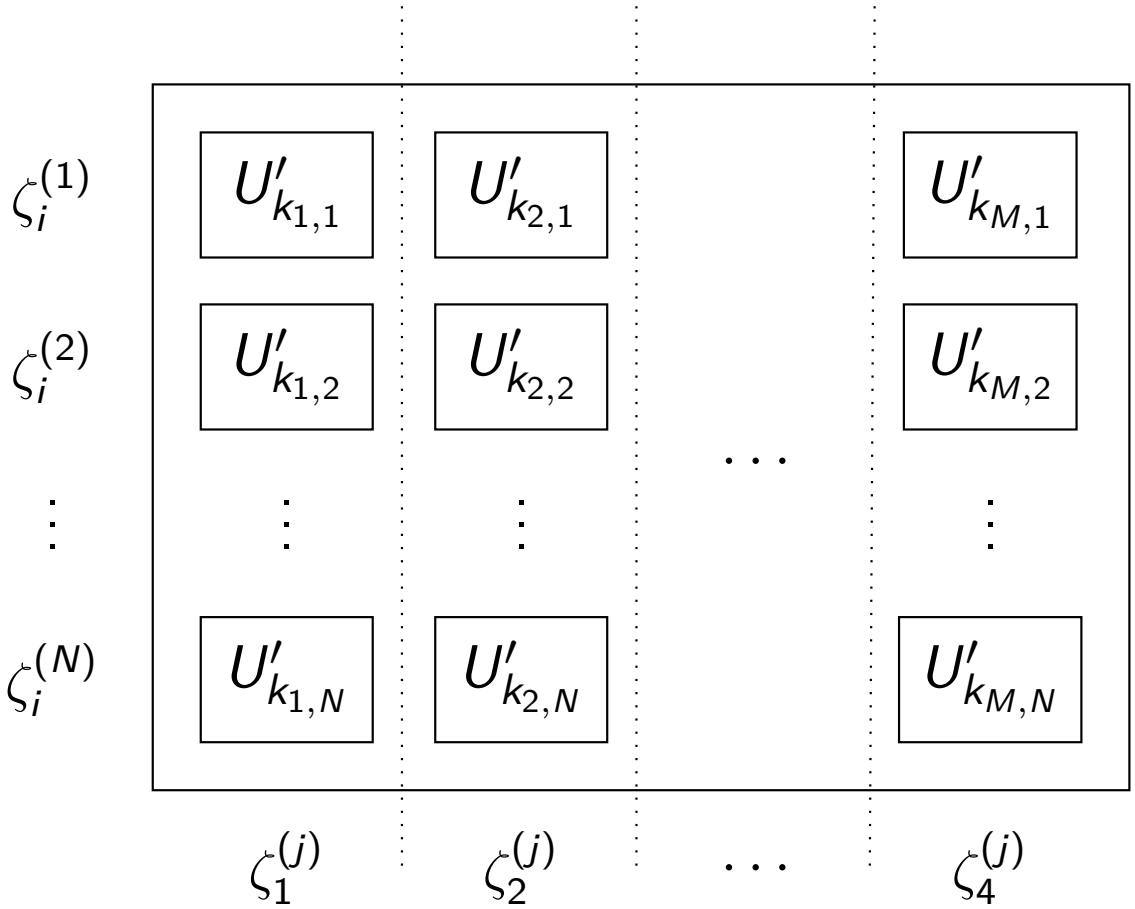

Figure: Parallel simulator

Contributions: Lemma 3

Considering that:

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$\rightarrow$ Effective query to $\mathcal{V}$ = request that actually happened **at runtime**.

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$$W = |1\rangle\langle 1| \otimes U_i + |0\rangle\langle 0| \otimes \mathbb{1}$$

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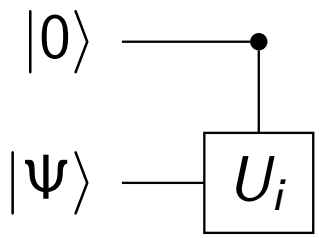

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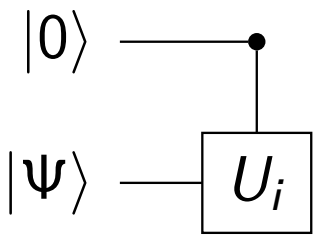

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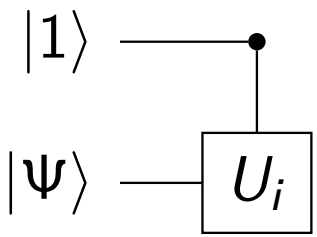

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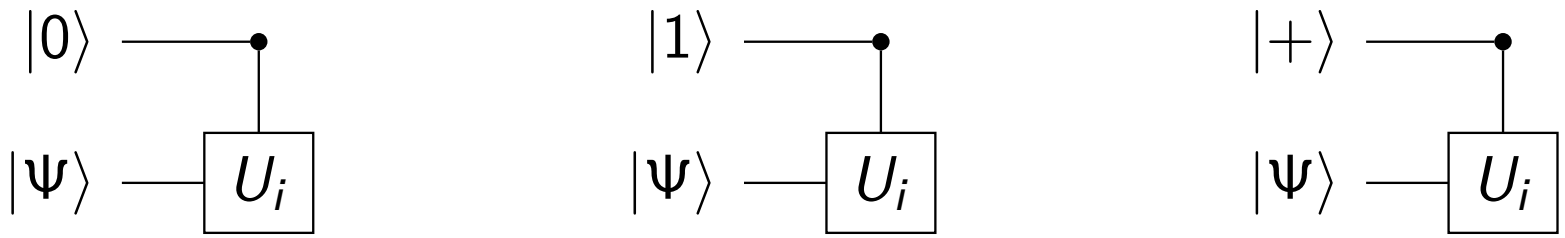

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Figure: Some variations of the control qubit in W

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

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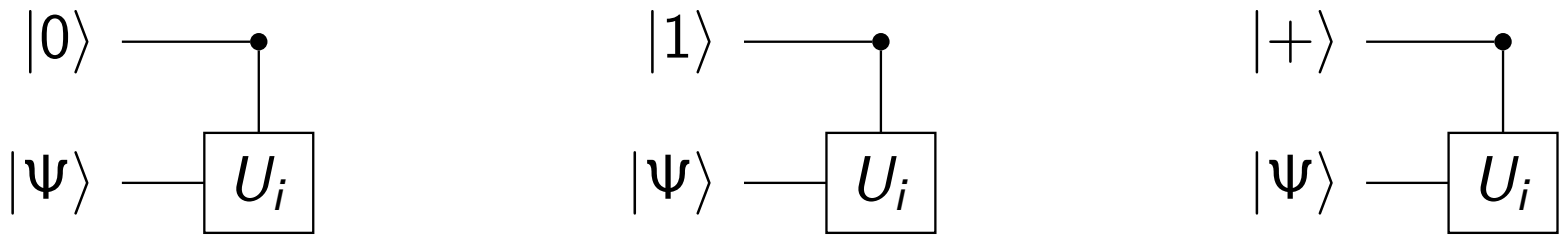

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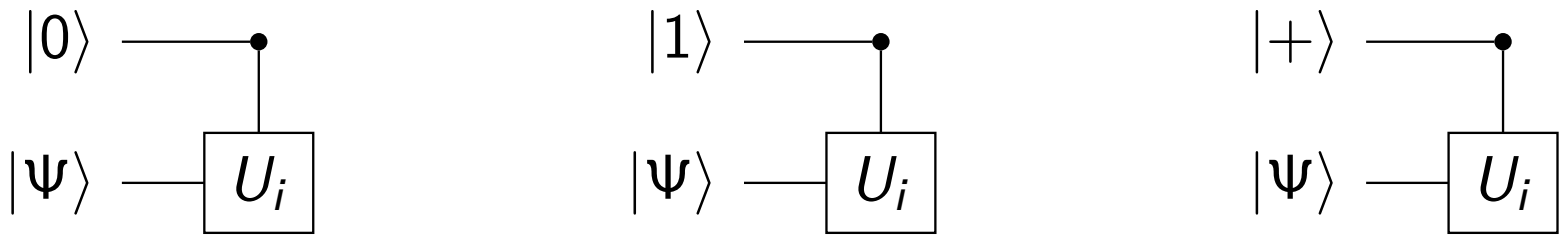

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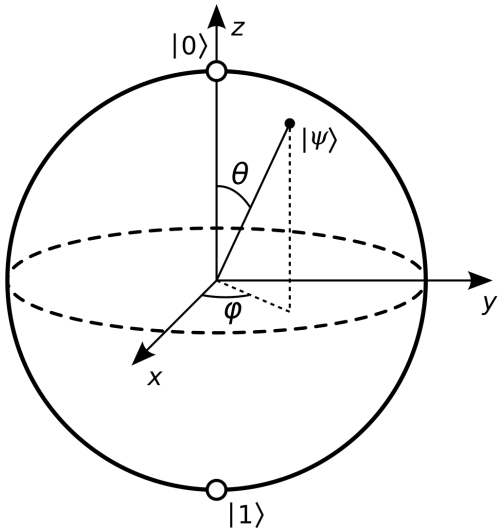
1. Prove that Lemma 3 implies the existence of Sim_4, which makes a constant number of queries to $\mathcal{V}$ (without 'effective').
2. Prove that Sim_4 implies $\mathbb{L} \in \mathbf{BQP}$ (see [Chi+21])

Thank you for your attention



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[Chi+21] Nai-Hui Chia et al. *On the Impossibility of Post-Quantum Black-Box Zero-Knowledge in Constant Rounds*. 2021. arXiv: 2103.11244 [cs.CR]. URL: https://arxiv.org/abs/2103.11244.

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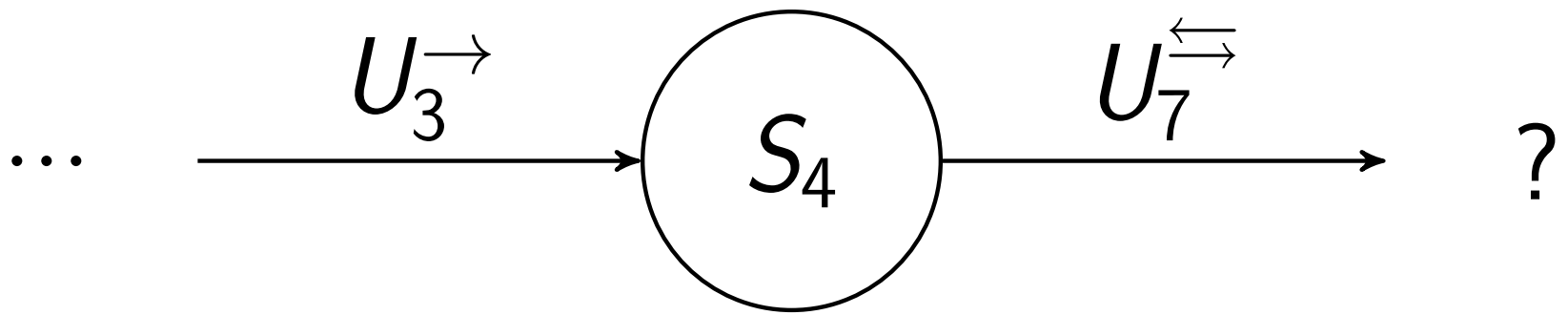

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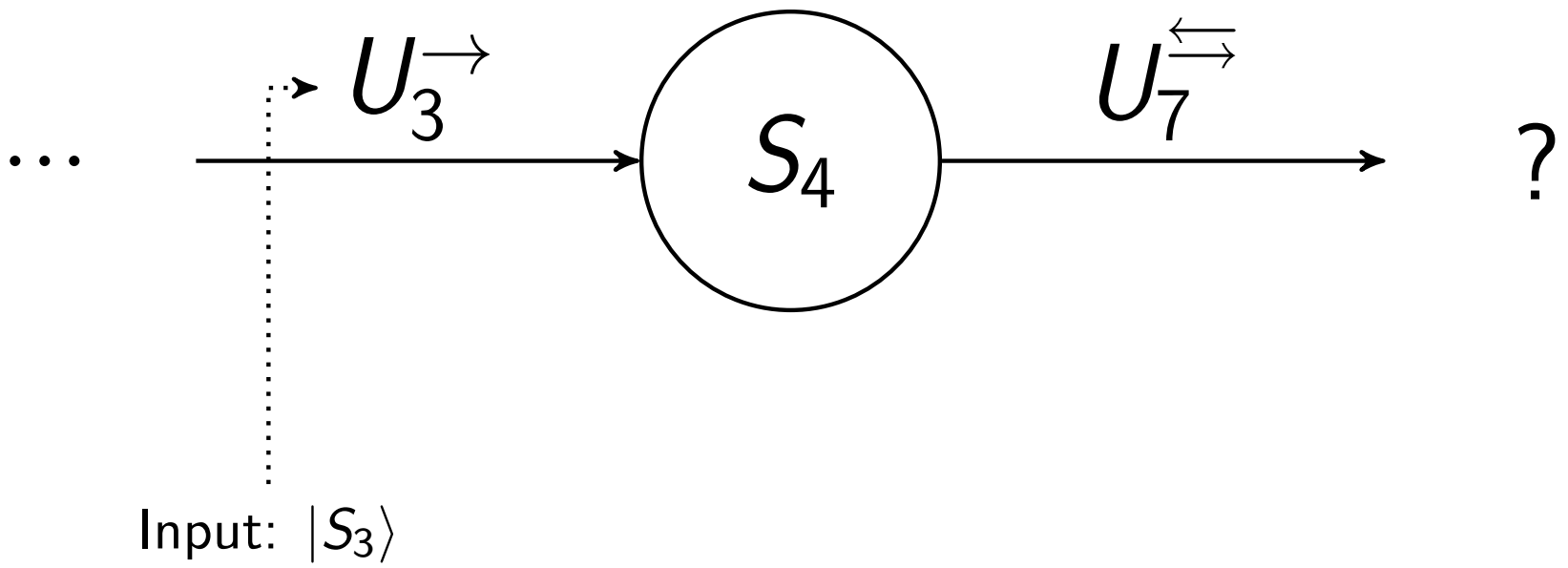

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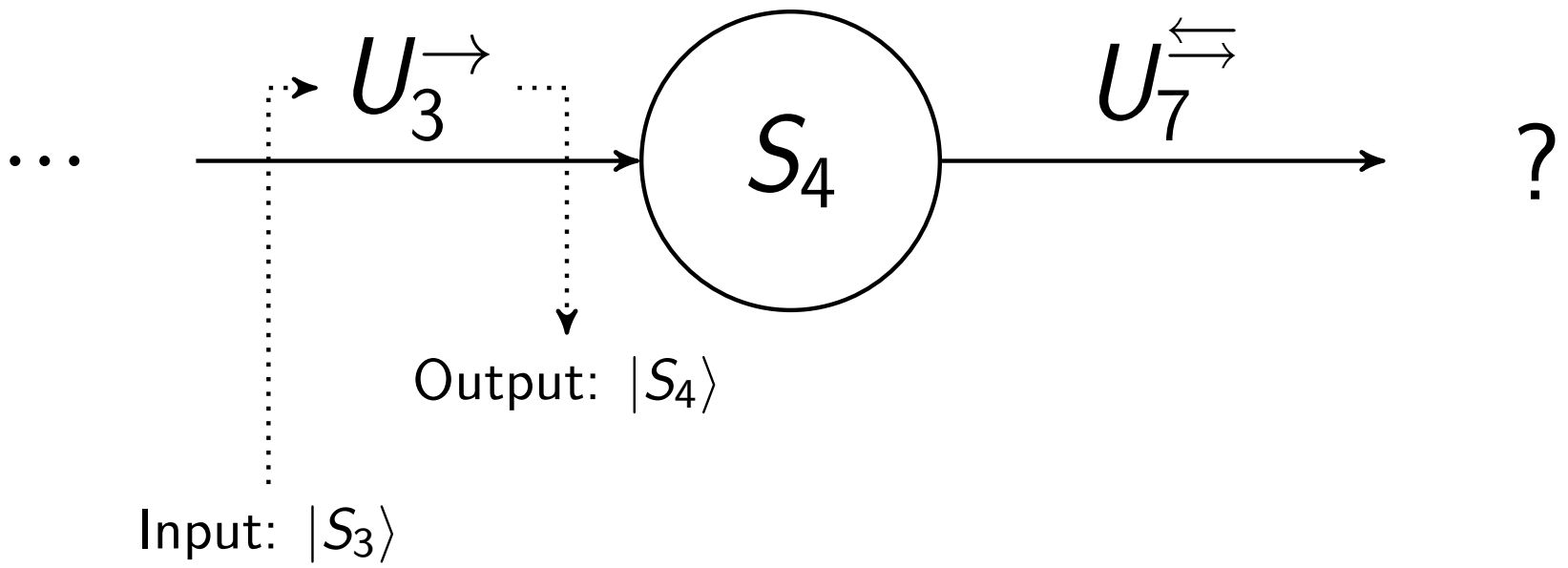

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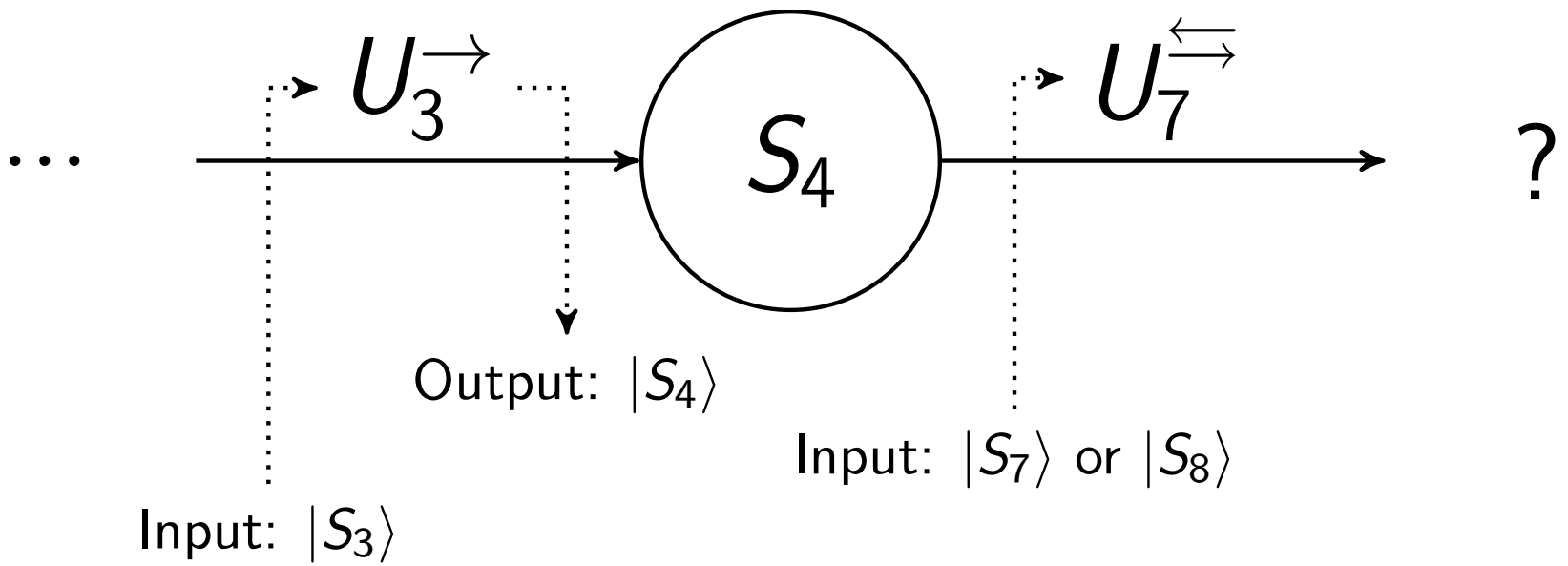

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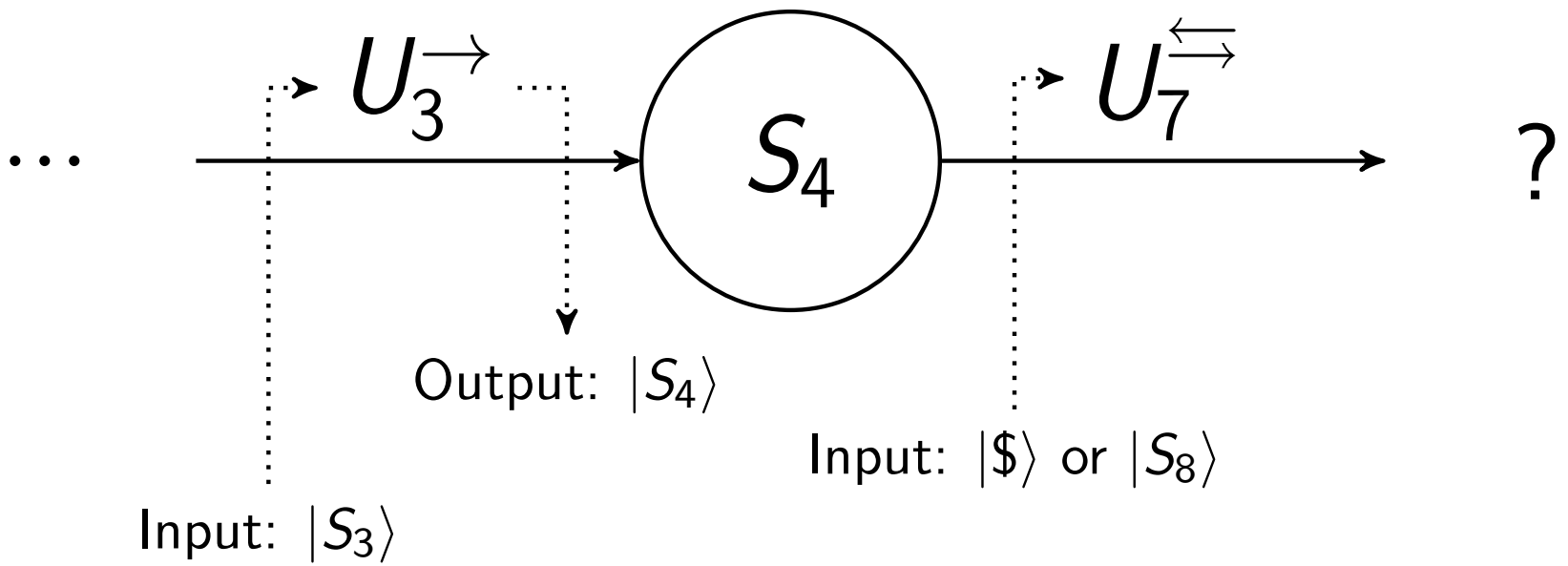

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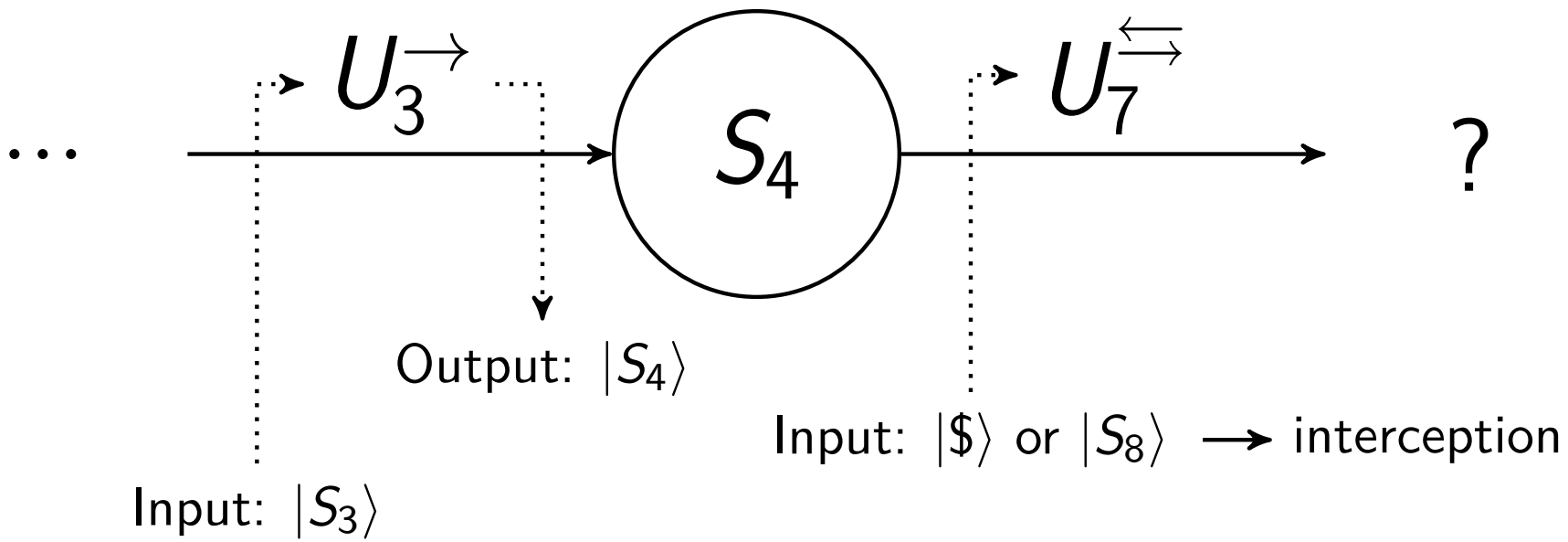

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